Section X2

First-Order Linear Differential Equations

Definition

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Solution

$$ye^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}dx + C$$

with an integrating factor of $u(x) = e^{\int P(x)dx}$

proof?

$u(x) \to IF$

$$\frac{d(u(x)\cdot y)}{dx} = u'(x)y + u(x)y'$$

$$\cancel{u(x)y'} + u(x)P(x)\cancel{y} = u'(x)\cancel{y} + \cancel{u(x)y'}$$

$$P(x) = \frac{u'}{u}$$

$$\int P(x) = \ln_e u$$

$$e^{\int P(x)\,dx} = u(x)$$

$xy' - 2y = x^2$

$$\frac{dy}{dx} - \underbrace{\left(\frac{2}{x}\right)}_{P(x)} y = \underbrace{x}_{Q(x)}$$

$$e^{\int(-2/x)dx} = e^{+\ln x^{-2}} = \boxed{x^{-2} = IF}$$

$$x^{-2}(y') - \frac{2}{x^3}y = \frac{1}{x} \qquad x^{-2}\cdot y$$

$$\frac{d(u(x)\cdot y)}{dx} = \frac{1}{x}$$

$$\frac{1}{x^2}y = \ln x + c$$

$$y = x^2[\ln x + c]$$

$\ln x$

$$y' - y\tan t = 1 \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

$$\frac{dy}{dt} \underbrace{-\tan t}_{P(t)} \cdot y = \overset{Q(t)}{1}$$

$$\int -\tan t \, dt = \ln|\cos t| \to IF: \cos t$$

$$\int -\frac{s}{c}$$

$$\underbrace{\cos t \cdot y' - \cos t \tan t \, y}_{\text{deriv of } \cos t \cdot y} = \cos t$$

$$\cos t \cdot y = \sin t + C$$

$$y = \tan t + C \cdot \sec t$$